Valuing human life and the environment: the J-value framework

to assess how much to spend on a protection system

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Human life

Weights and Measures

We load a heaped tablespoon of
'Just Roasted' beans into a single Pret cup.
We weigh one in three cappuccinos
on our special barista scales.
We do this to check the amount of
air in our Rachel's organic milk foam.
It's a velvety texture we're after.
The bubble size is crucial.
The smaller the bubbles, the
more delicious the drink.
Obsessive but important.

PRET PASSION FACT NO. 94

The crucial variable

- Pret's coffee model has identified bubble size as the **crucial variable for texture and flavour**.
- **Measurement and control** give Pret's cappucinos **consistency** – *consistently good in my opinion!*
- But in safety analysis and discussions, there has long been confusion as to what the crucial variable should be.
- Establishing the **crucial variable for safety** is the first step to managing safety.

The mathematics of safety

- The term "a human life" is in common use as a basis for valuation. But this binary variable is too imprecise for our purposes.
- There is also confusion about what it means, both in political and scientific discourse.
- As an example, we cannot save anyone's life.
- To be sure, you can save someone's life from an immediate threat, and therefore on a temporary basis. But it is important to know how long the life will stay saved.
- For example, saving a condemned man from drowning a day before he is due to be executed – how much will he thank you for that? How much should he thank you?

Life expectancy is the crucial variable in safety analysis

- **Life to come** is what is valuable to each of us and it is only this that can form a rational basis for a calculus of safety. But life to come is a **random variable**, for which life expectancy is our best estimate.
- Going back to "life saving", the best we can do is avert a current threat and restore that person's **life expectancy** to what it was before.
- The importance of life expectancy was realised by
  - Lord Marshall *et al*: *Big nuclear accidents*, 1983.
  - Nathwani and Pandey, who used it as a major component in their Life Quality Index: *A conceptual approach to the estimation of societal willingness-to-pay for nuclear safety programs,* 2003.

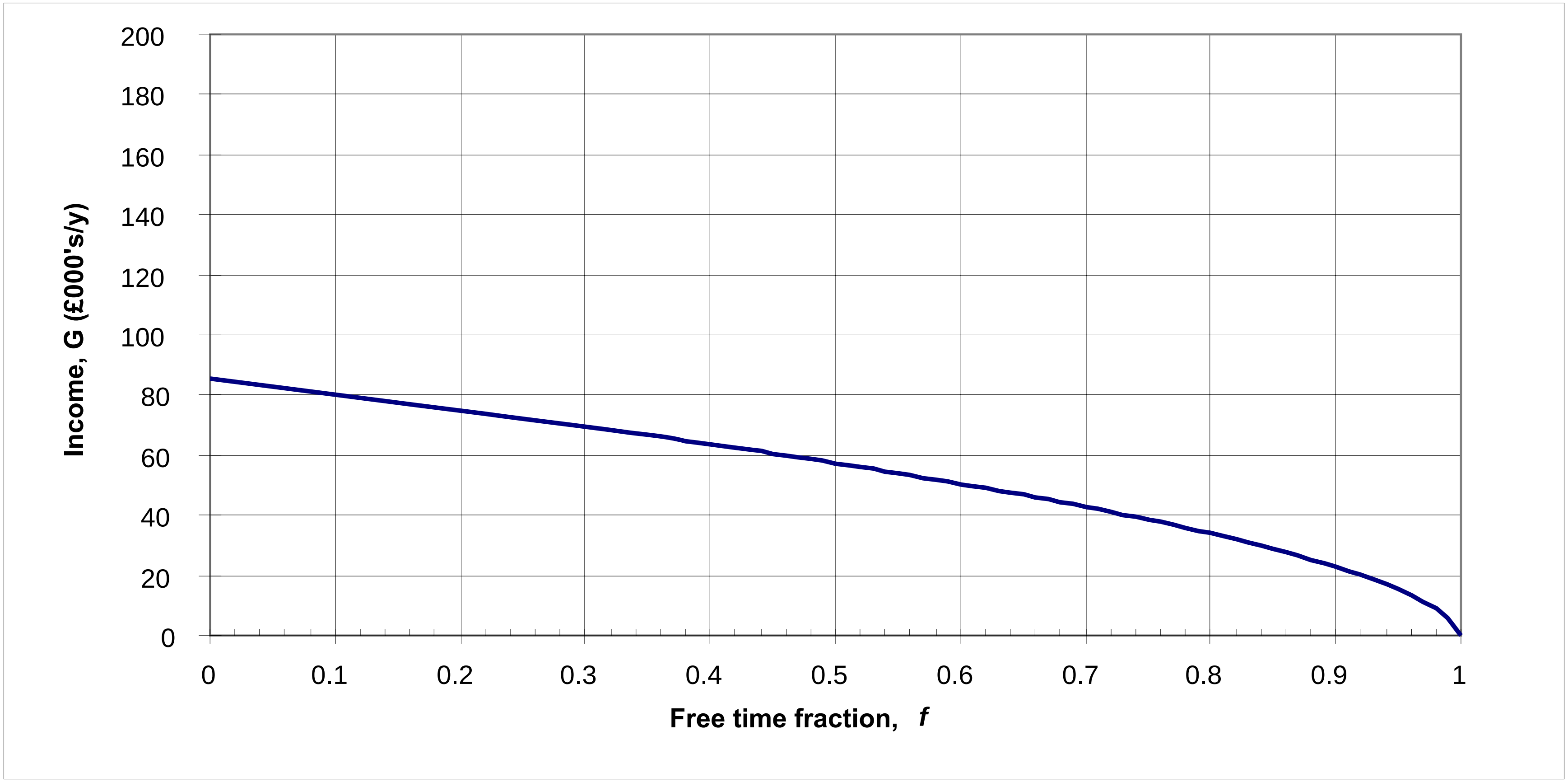
The link between safety and economics

- Any system can be made safer (= extend life expectancy) by spending more money on a further protection system. So a **trade-off** must be made, **always**. Safety and economics are linked inextricably.
- In fact, **2 trade-offs** are made when deciding whether or not to install extra protection.
- The first is made at a societal level, and is between free time and income.

The 1st trade-off

- Life quality is assumed to depend on
  - **your income**,
  - **time that is yours to do with as you please: your free time**
- It will be found then to depend also on the appropriate value of the Pratt-Arrow risk aversion parameter.
- The life-quality index will be optimised, subject to the constraint of income versus free time.
- The average income is modelled by a Cobb-Douglas Production Function, which accounts for capital as well as wages.
- The properties of the resulting trade-off enable us to define the "life risk-aversion", the value of Pratt and Arrow's coefficient of relative risk aversion applicable to valuing human life expectancy.

1^{st} Trade-Off: Income vs. Free time fraction from now on (UK data 2007)



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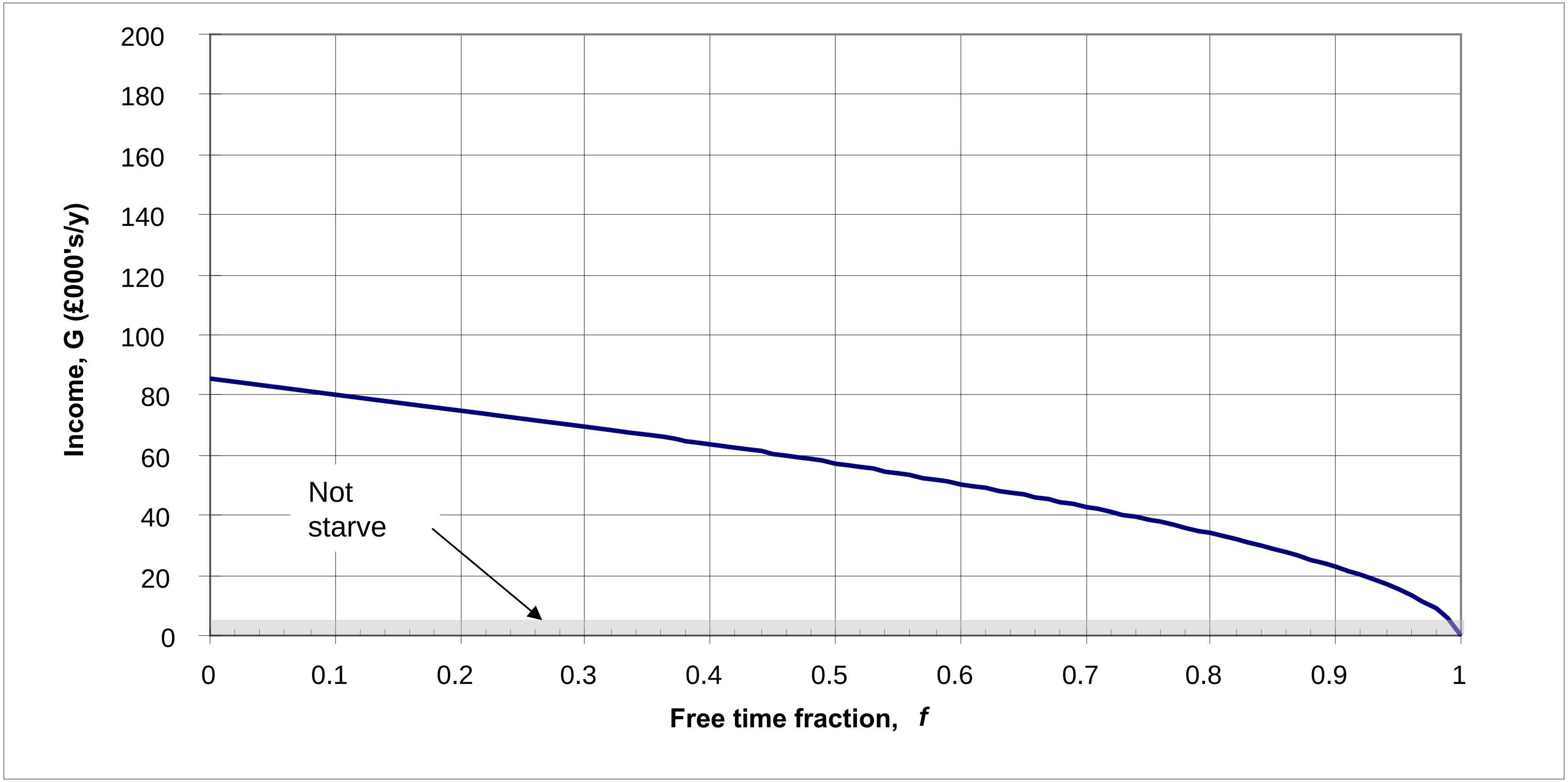
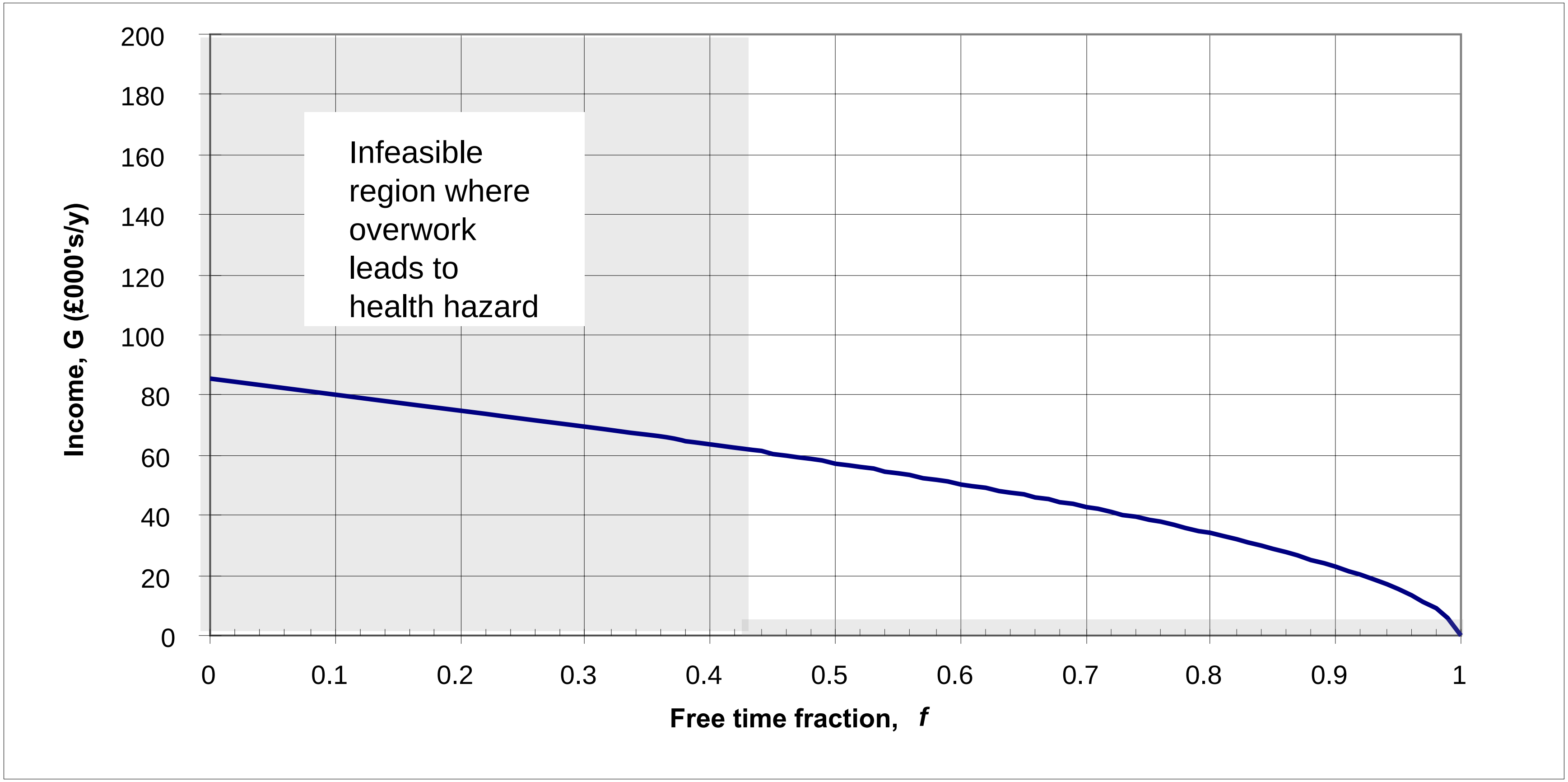
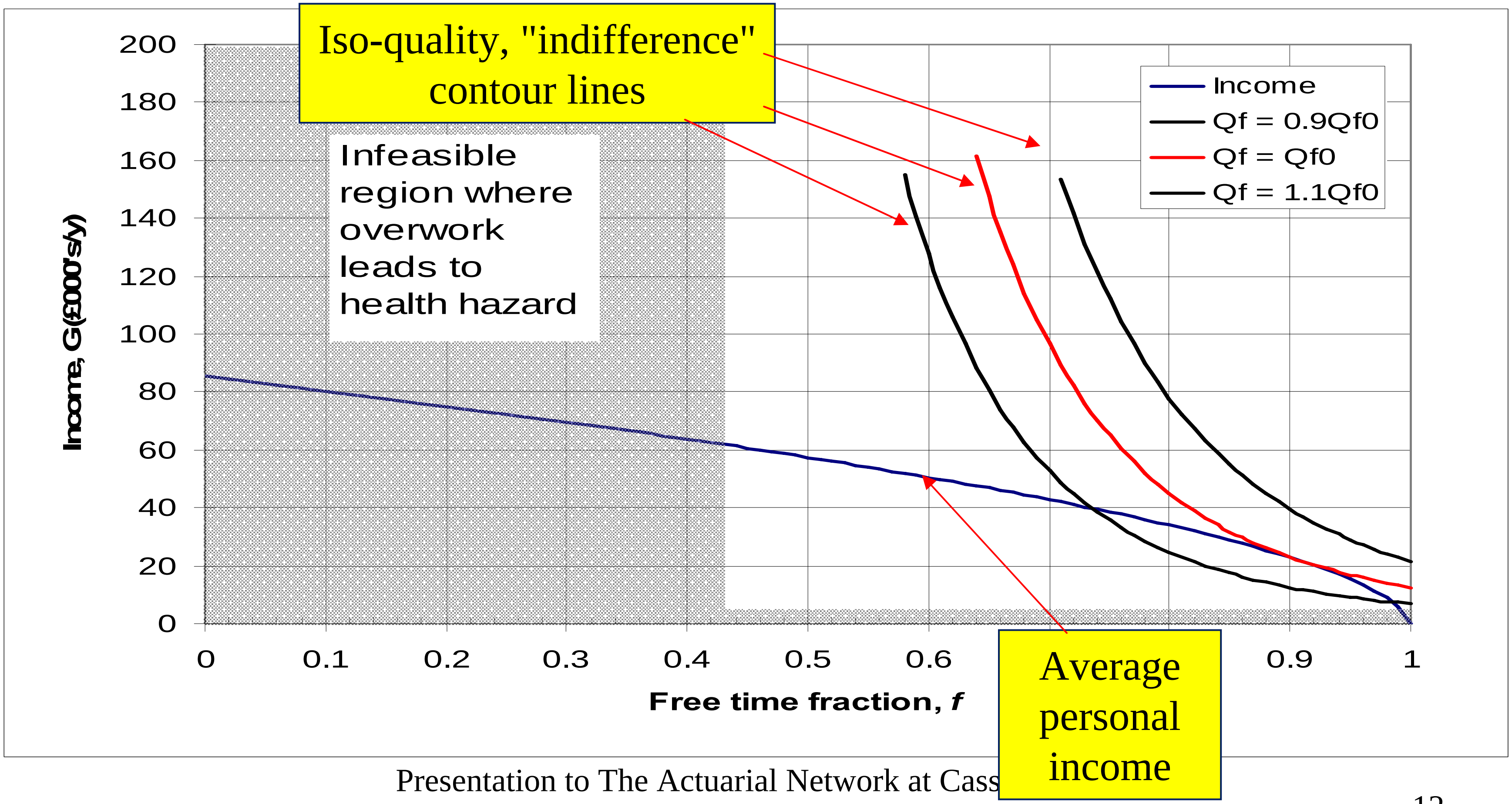


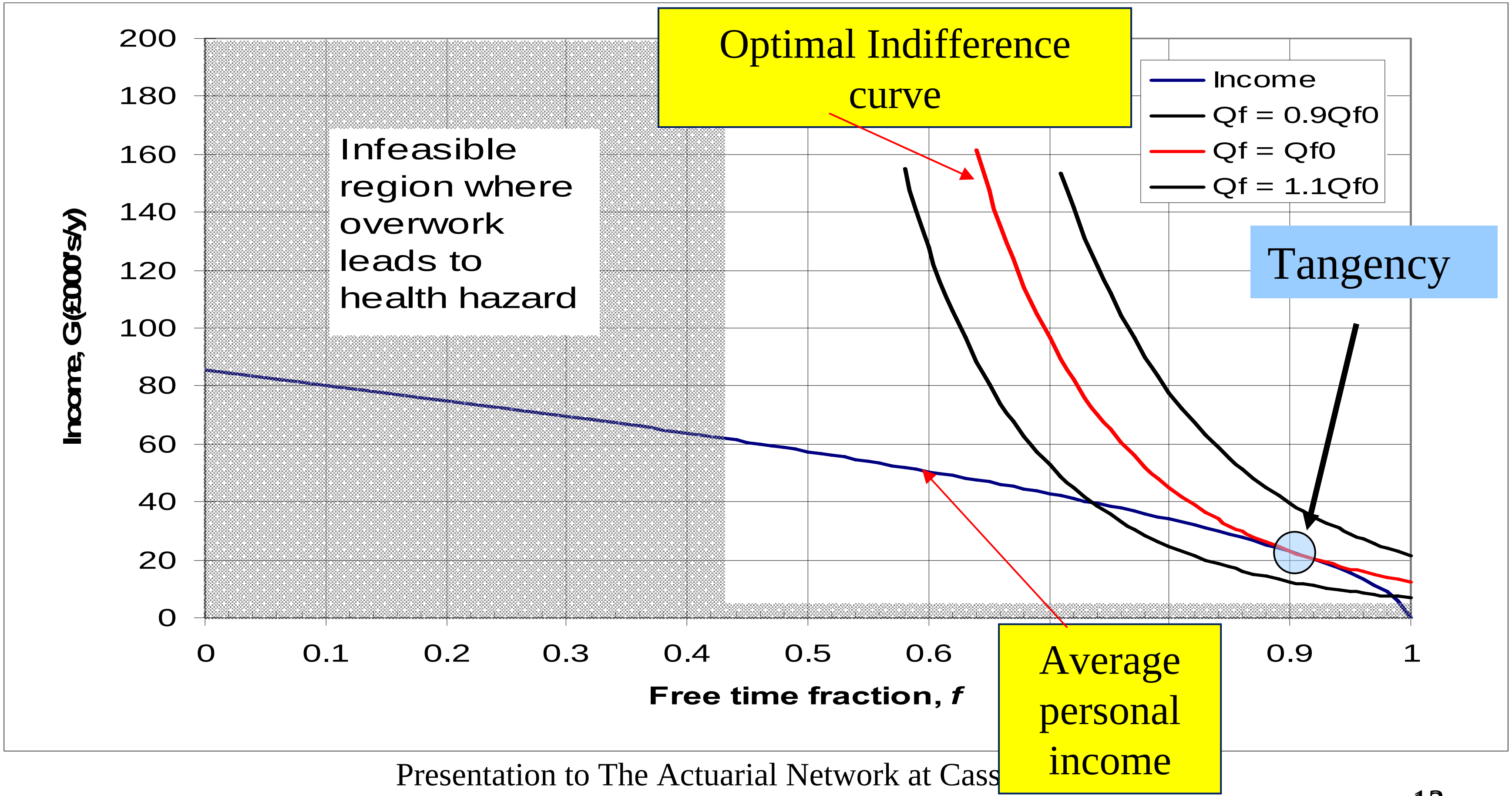
Income & Free Time Fraction Trade-Off



Income & Free Time Fraction Trade-Off



Income & Free Time Fraction Trade-Off



Matching derivatives in the blue circle, we find an expression for life risk-aversion, ε

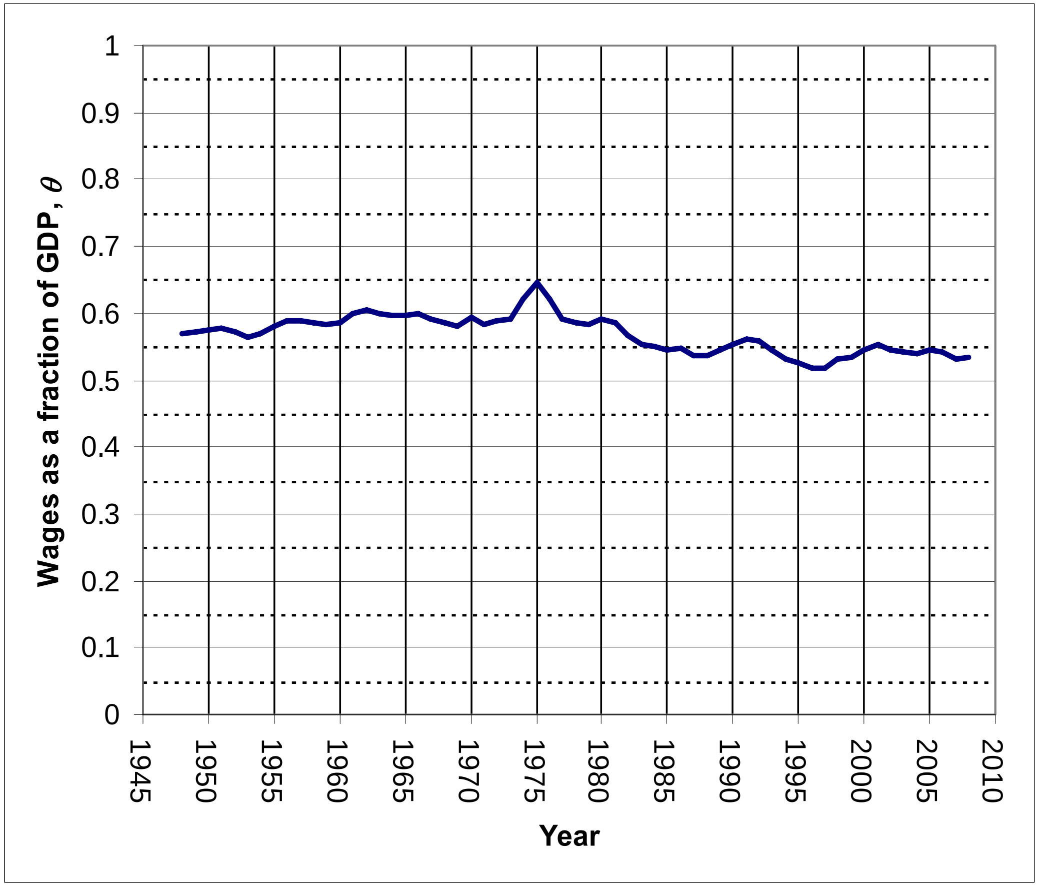
(the Pratt-Arrow coefficient of relative risk aversion)

$$\varepsilon = \frac{1 - \frac{\theta + 1}{\theta} w}{1 - w}$$

w = average working time from now on,

θ = share of wages in Gross Domestic Product (GDP)

Working time fraction to the end of life against age (UK).
Red line = average across people of all ages = ***w* = 0.091**



Wages as a fraction of GDP, θ

$\theta = 0.546$, $\sigma = 0.018$ over last 30 years, UK

Calculation of life risk-aversion, for use in valuing life extension

$$\varepsilon = \frac{1 - \dfrac{\theta + 1}{\theta} w}{1 - w} = 0.82$$

(UK)

The 2nd trade-off when considering a protection system

- Suppose the person being protected has to contribute to the cost of the safety system.
- The test is whether his **decrease in utility** from the fall in income will be matched by a sufficient **increase** in his life expectancy. He will spend as long as his life quality rises. He will stop spending when his life quality falls.
- In practice someone else will normally be paying, but this is a form of the **Hicks-Kaldor compensation principle** which states that the gainers (eg the factory) should be making a sufficient gain to be able to compensate the losers (workers and public). While Hicks-Kaldor does not enforce the compensation, here the compensation is effectively paid.

The Life-Quality Index, Q

$$Q = G^{1-\varepsilon} X_d$$

$G =$ average income (£/y),

$X_d =$ (discounted) life expectancy

$\varepsilon =$ life risk - aversion

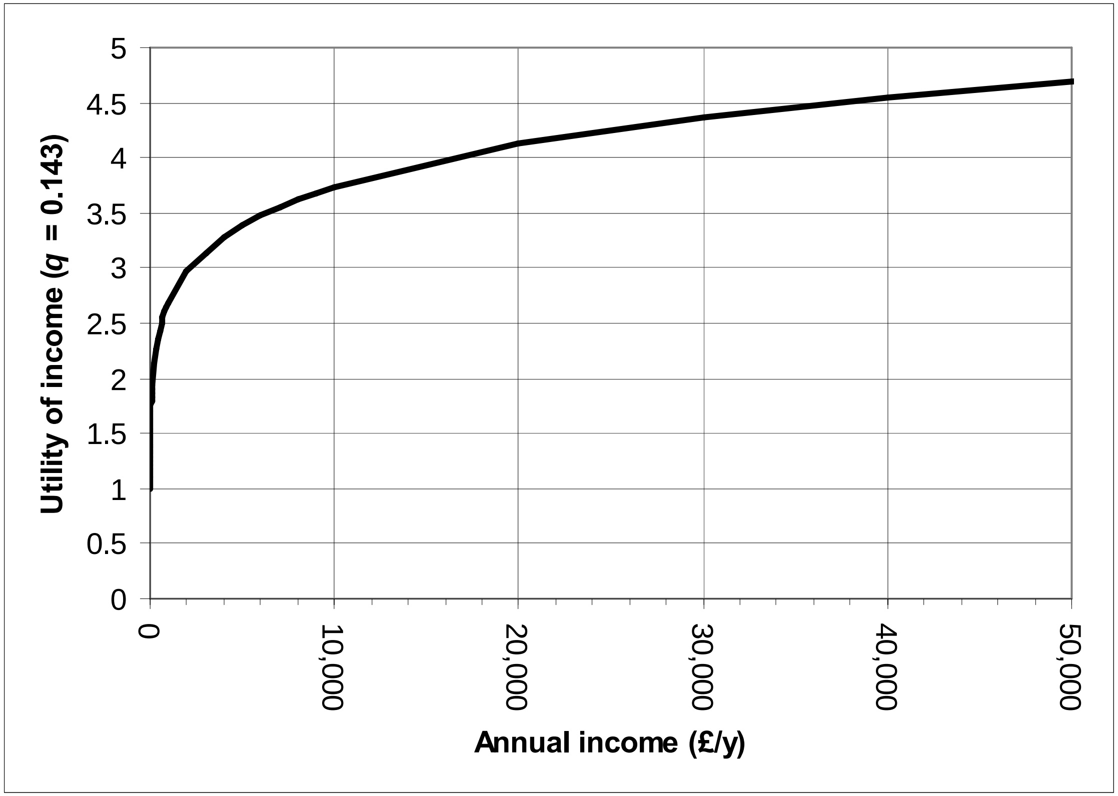
$G^{1-\varepsilon}$ is the utility of the income

Utilities in future years may be discounted at about 2.5% p.a. :

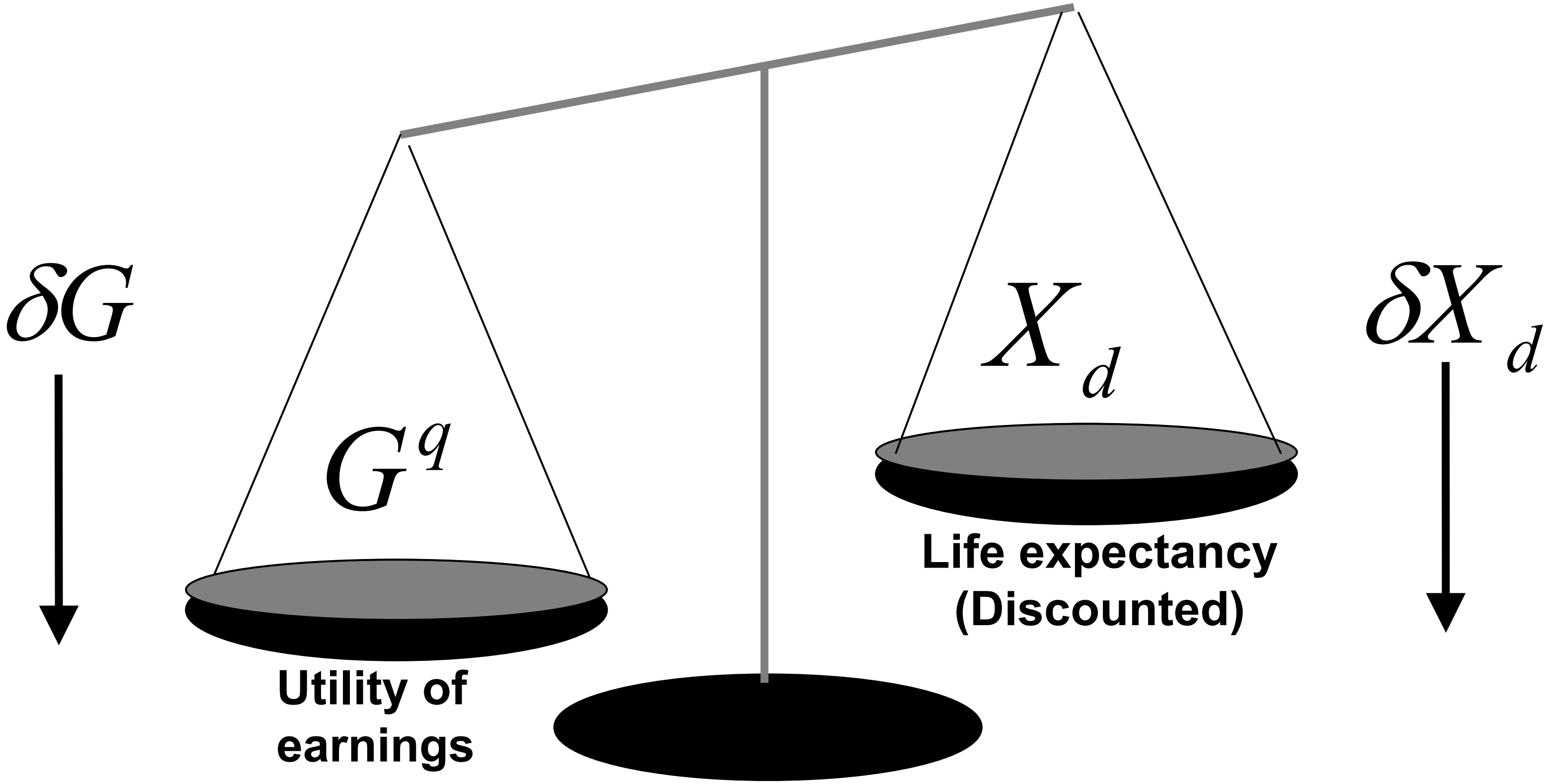
"Jam today" is worth more than jam tomorrow

- Lewis Carroll, *Through the looking glass*, 1871

The utility of earnings will decrease as the earnings increase (the 3rd Ferrari matters less!).

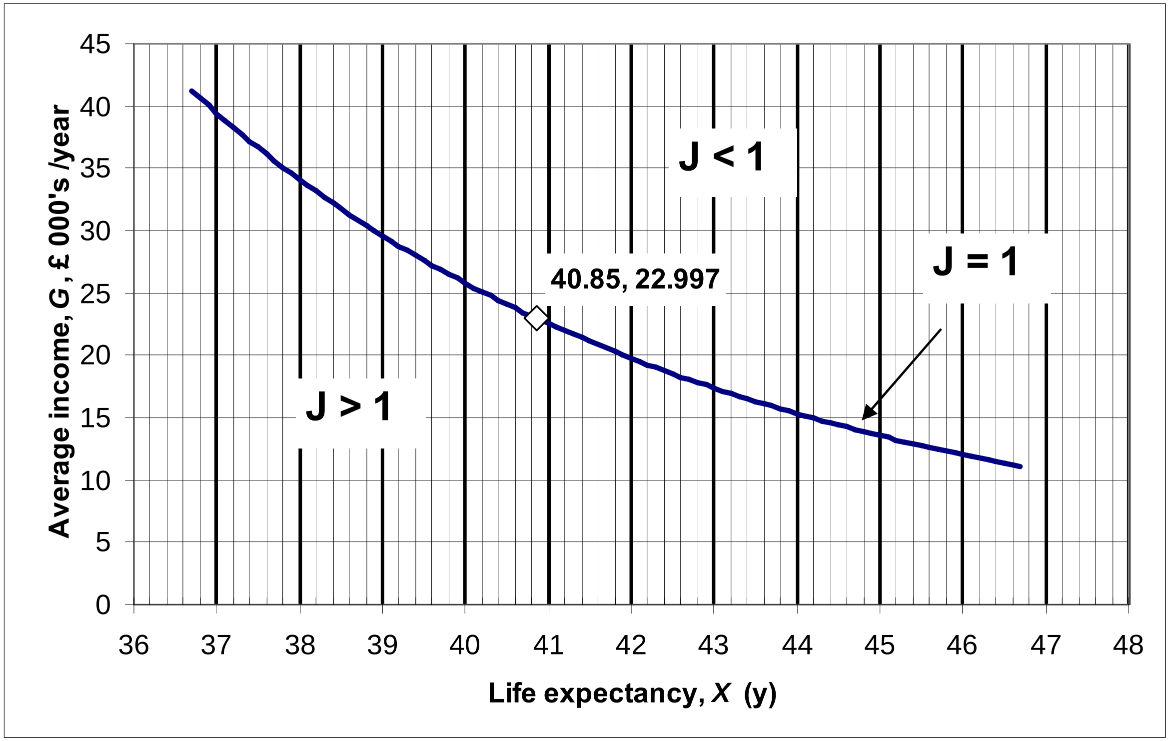


Balance occurs when δQ = 0, J = 1: no change in life quality index



2^{nd} trade-off, when considering a safety system

J = 1 indifference curve (UK, 2007 data) – J for Judgment



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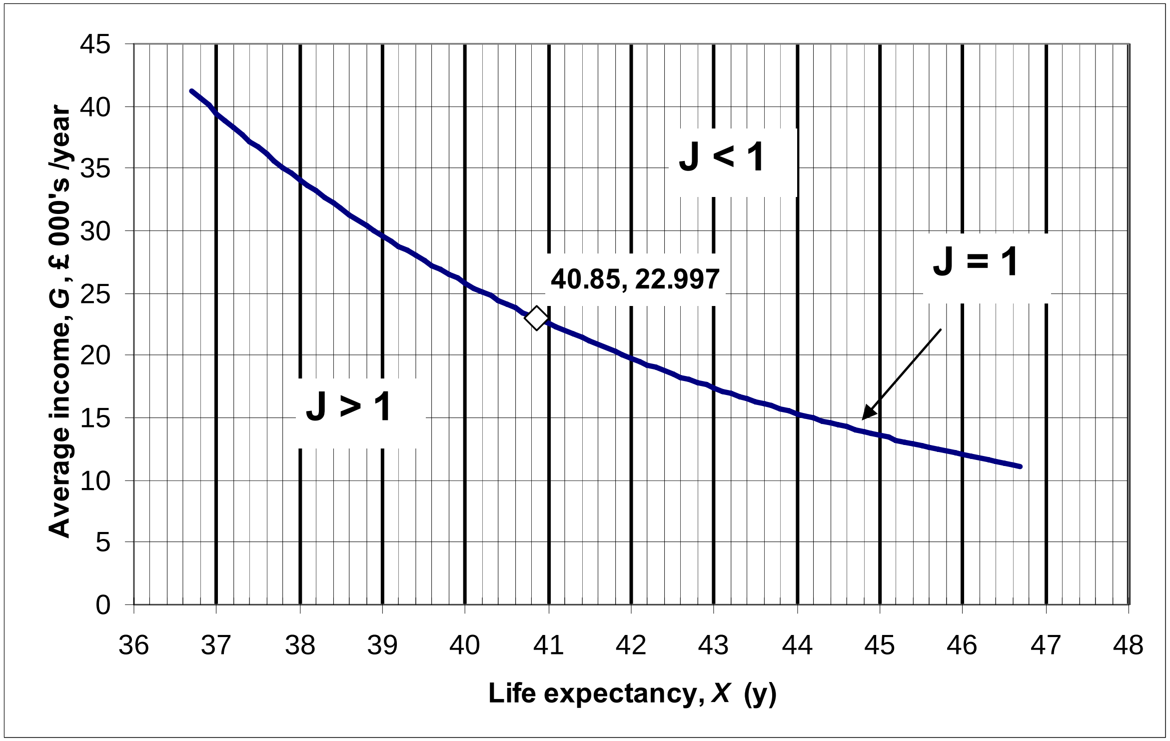


"A merry life and short one shall be my motto", Black Bartholomew Roberts, Welsh pirate, (1682 – 1722)

The maximum that can fairly be spent: on an annual basis

$$-\delta G_N(t) = \frac{NG}{1-\varepsilon}\frac{\delta X_d}{X_d} \qquad \text{for } 0 \le t \le X_d$$

$N =$ number of people in affected cohort

δX_d = average change in discounted life expectancy from safety measure

$\delta G_N(t) =$ maximum annual payment,

X_d = average discounted life expectancy across the cohort

The maximum that can fairly be spent as an up-front lump sum, δV_N

$$\delta V_N = \frac{1 - e^{-r_d X_d}}{r_d} \delta G_N$$

or

$$\delta V_N = \frac{1 - e^{-r_d X_d}}{r_d X_d} \frac{NG}{1 - \varepsilon} \delta X_d$$

where δV_N is the maximum single, up-front capital payment that is reasonable and $r_d =$ discount rate.

The J-value

$$J = \frac{\delta \hat{V}_N}{\delta V_N}$$

$\delta \hat{V}_N$ is the actual equivalent lump-sum spend

δV_N is the maximum reasonable equivalent lump-sum spend

Calculating life expectancy

- We use actuarial life tables to calculate the life expectancies before and after the safety measure.
- This is done by changing the hazard rate in life tables.
- Delayed effects of radiation are more difficult, but we have built on the framework devised by Lord Marshall.

The J-value is objective

- The J-value depends on:
  – the cost of the safety scheme
  – the size of the benefiting group
  – their average income
  – before and after life expectancies
  – the working time fraction from now on
  – the share of wages in GDP
  – the long-term discount rate (0%, 2.5%, 4% p.a.)

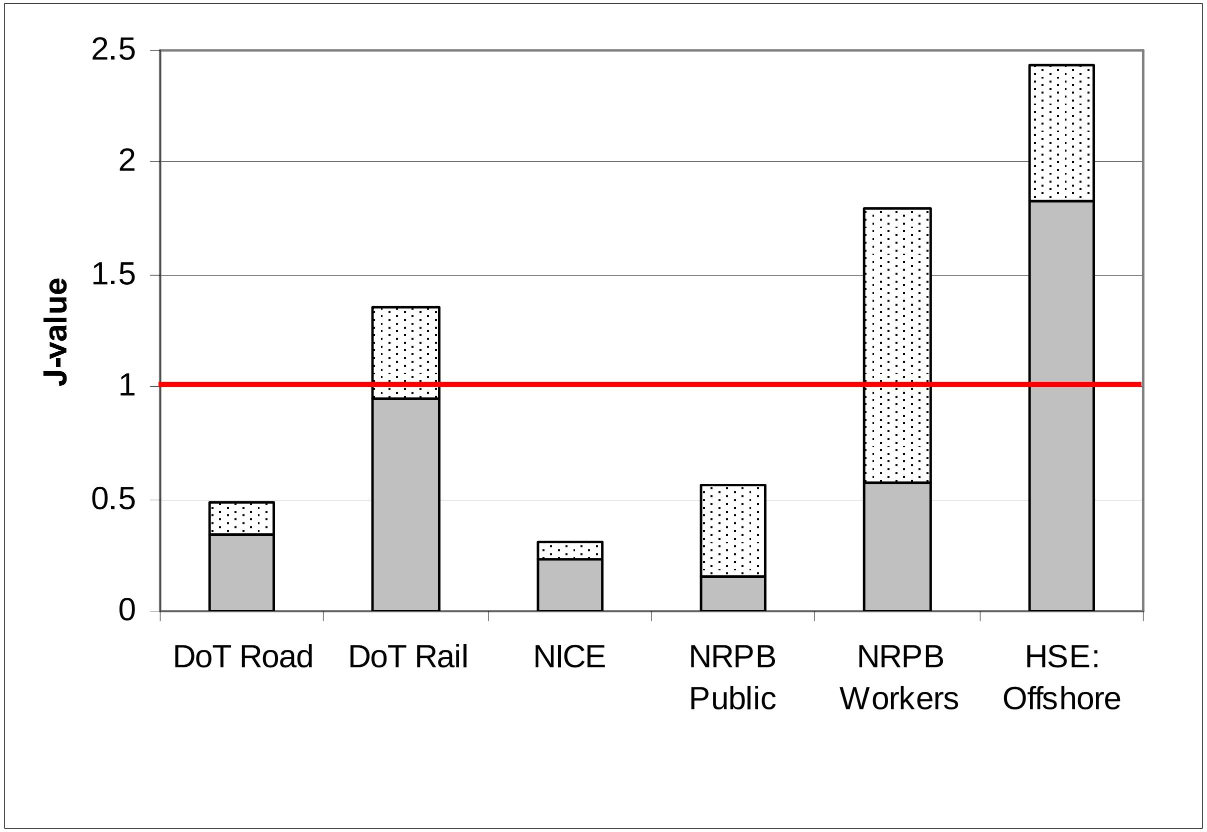
Specimen J-values and their meanings:

the lower the J-value the stronger the motivation to sanction the safety system

- J = 0.1: the safety spend is acceptable: the Life Quality Index goes up and society receives a good net benefit.
- J=1.0: the safety spend is on the limit of acceptability: the effect on society is neutral. This is the risk-averse but still reasonable position.
- J=2.0: the safety spend is unacceptable, imposing net disbenefits on society. The spend would need to be halved to be acceptable.

Regulators' Recommendations

(ranges depend on calculation method and discount rate, 0% or 2.5% pa)



Case Studies:
Reality versus theory

Case study 1: J-values for Railway Protection Systems

TPWS	11.3
ERTMS	138

Comment: both high, ERTMS by a very large factor

TPWS has been installed (2003), ERTMS has not.

Case study 2: Petrol-forecourt emissions:
Petrol delivery

Volatile Organic Compounds (VOC's) control system for smaller petrol stations	2.4 (but with large uncertainty)

Derogation from VOC control regulations granted for small petrol stations

Case study 3a: J-values for NICE decisions on breast cancer

Vinorelbine for metastatic breast cancer	0.014
Paclitaxel for advanced breast cancer	0.046
Docetaxel for advanced breast cancer	0.045

All recommended by NICE

Case study 3b: J-values for NICE decisions

Zanamivir for influenza	0.016
Imatinib for chronic myeloid leukaemia	0.68

NICE has not recommended Imatinib

Case Study 4: J-values for BSE/vCJD countermeasures

Early countermeasures (up to 1990)	0.37
Post-1996 countermeasures	368

Comment: early countermeasures sensible, later countermeasures very clearly not reasonable.

Case study 5: J-value for BNFL's Technetium-99 Removal Plant

Technetium-99 Removal Plant: Critical group of 2663 people	184

- The J-value should be 1.0 or less, so that an overspend of 2 orders of magnitude occurred.
- The safety system was calculated to extend the life expectancy of the average member of the critical group by 3 hours.

Conclusions for safety analysis

- Life expectancy is a well-documented and regularly updated variable that can form the basis of a rigorous safety calculus.
- The J-value method uses measured, economic and actuarial data to provide an evaluation of the human life extension achieved by a safety scheme that is **wholly objective** and **fully transparent.**
- The J-value method offers the possibility of **consistency** in decisions about safety for the first time.

Extension to the environment

- As civilisation advances, we want *not only* increasingly high levels of safety for humans *but also* **greater protection for the environment.**
- The problem is that, up to now, no one has been able to give an objective answer on how much should reasonably be spent on protection.
- But a fully objective answer is again possible.

The Environment

- We will characterise all environmental damages by their costs.
- This will allow all other costs (loss of business, business disruption after a large industrial accident etc.) to be subsumed here also.

The J-value Framework

- The **J-value** shows how much should be spent to protect against human harm.
- The **J_2-value** shows how much should be spent to protect against environmental and other costs.
- The **J_T-value** combines the results and indicates how much should be spent on a protection system.
- $J_T = 1$ indicates the maximum, sensible spend, and $J_T > 1$ means an overspend.
- J_T, like J and J_2, is fully objective

- As with the J-value, the J_2-value is derived using Utility Theory, although using a different route.
- The utility function used is an Atkinson Utility Function, first proposed by Sir Tony Atkinson, as a variant of the Power Utility, which is used in the derivation of the J-value.

Allowing for environmental and all other costs:

J_2-value
J_T-value

Partition the protection system spend

$$\delta\hat{Z} = \delta\hat{W} - J^{*}\delta V_{N}$$

$\delta\hat{W}$ is the total actual cost of protection system,

$\delta\hat{Z}$ is the residual, assumed to be spent to guard against environ
- mental costs

J^{*} is a disproportion factor to be applied to δV_{N}
the maximum reasonable to spend on protecting
the cohort of N people. (There are good grounds for setting $J^{*} = 1$,
as will be shown at the end.)

The J_2-value

$$J_2 = \frac{\delta \hat{Z}}{\delta Z_R}$$

where δZ_R is the maximum amount that it is reasonable for the organisation to spend to guard against environmental costs, including a disproportion factor to take account of the size of the potential cost.

The J_T-value

$$J_T = \frac{\text{What is actually spent on the protection system}}{\begin{pmatrix}\text{The maximum that should be} \\ \text{spent on environmental protection}\end{pmatrix} + \begin{pmatrix}\text{the maximum that should} \\ \text{be spent on safety}\end{pmatrix}}$$

$$= \frac{\delta \hat{W}}{\delta Z_R + \delta V_N}$$

The appropriate value of risk-aversion, ε

It is found that the appropriate value of risk-aversion, ε, will depend on the assets of the company taking the decision and the size of the environmental costs.

For a risk averse company, the lowest value of ε is 0, while the highest will be $\varepsilon_{\max}$, a top value that is determined by the point of indiscriminate decision, as will be shown later.

Calculating δZ_R

$$\delta Z_R = M_R(\varepsilon_{\max})\delta Z_0$$

where δZ_0 is the amount that it would be sensible for the organisation to spend to protect against environmental costs at the risk-neutral value of risk-aversion, namely $\varepsilon = 0$. while $M_R(\varepsilon_{\max})$ is the disproportion or gross disproportion factor which gives the maximum sensible spend.

δZ_0

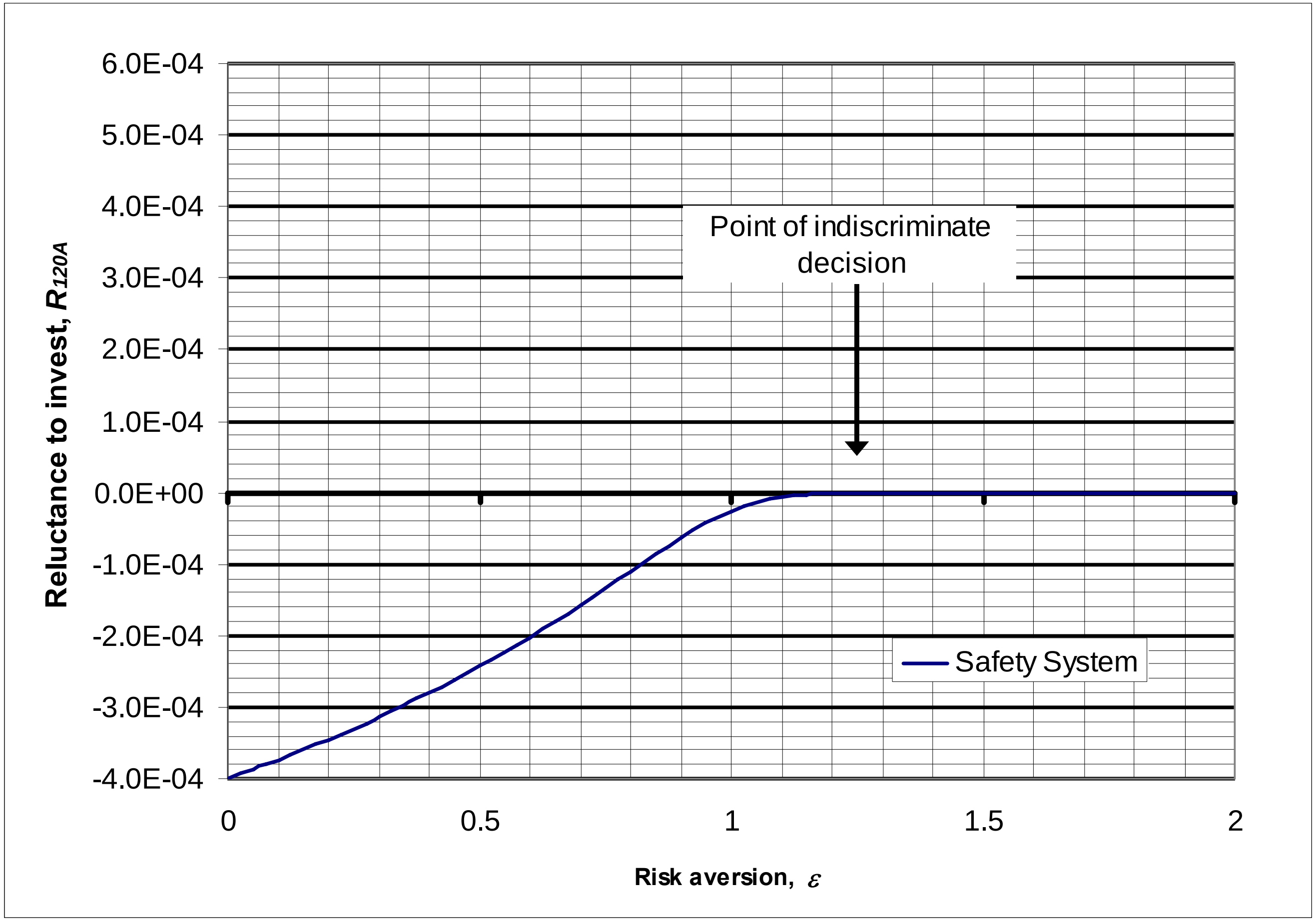
- This depends on
  - the frequency of the accident before the protection system is installed and the frequency afterwards
  - the cost of the accident should it occur
  - an allowance for the accident's expected time of occurrence within the operating period, should it occur
  - the growth rate of the organisation

$$M_R\left(\varepsilon_{\max}\right)$$

The maximum sensible value of the risk multiplier depends on finding the point of indiscriminate decision, the value of risk-aversion, ε, at which the organisation will be unable to discriminate between the advantages of installing the protection system or not.

- Utility theory allows us to develop a graph of the desire to invest (the negative of reluctance to invest) in a protection system against risk-aversion.
- As risk-aversion increases, so the desire to invest decreases for a protection deemed good value on monetary grounds alone, at $\varepsilon = 0$.
- The desire to invest will remain positive, but the incentive will decrease as risk-aversion increases.
- At some high value of risk-aversion, it will be impossible to discriminate between the advantages of the safety system and those of doing nothing. This is the **point of indiscriminate decision.**
- Effectively the decision maker has become so risk-averse that he does not even want to take a decision.

The point of indiscriminate decision



In fact, it is worse than this, because:

- as risk-aversion increases, so the reluctance to invest decreases for a protection system deemed very poor value on monetary grounds alone, at the risk-neutral point, $\varepsilon = 0$.
- There will always be a reluctance to invest in a very poor value system, but again, at some high value of risk-aversion, discrimination will be lost: a **point of indiscriminate decision** will occur again.
- At some point, therefore, the decision-maker is so risk-averse that he is effectively panicking, and not discriminate between installing a good system, a bad system or doing nothing.

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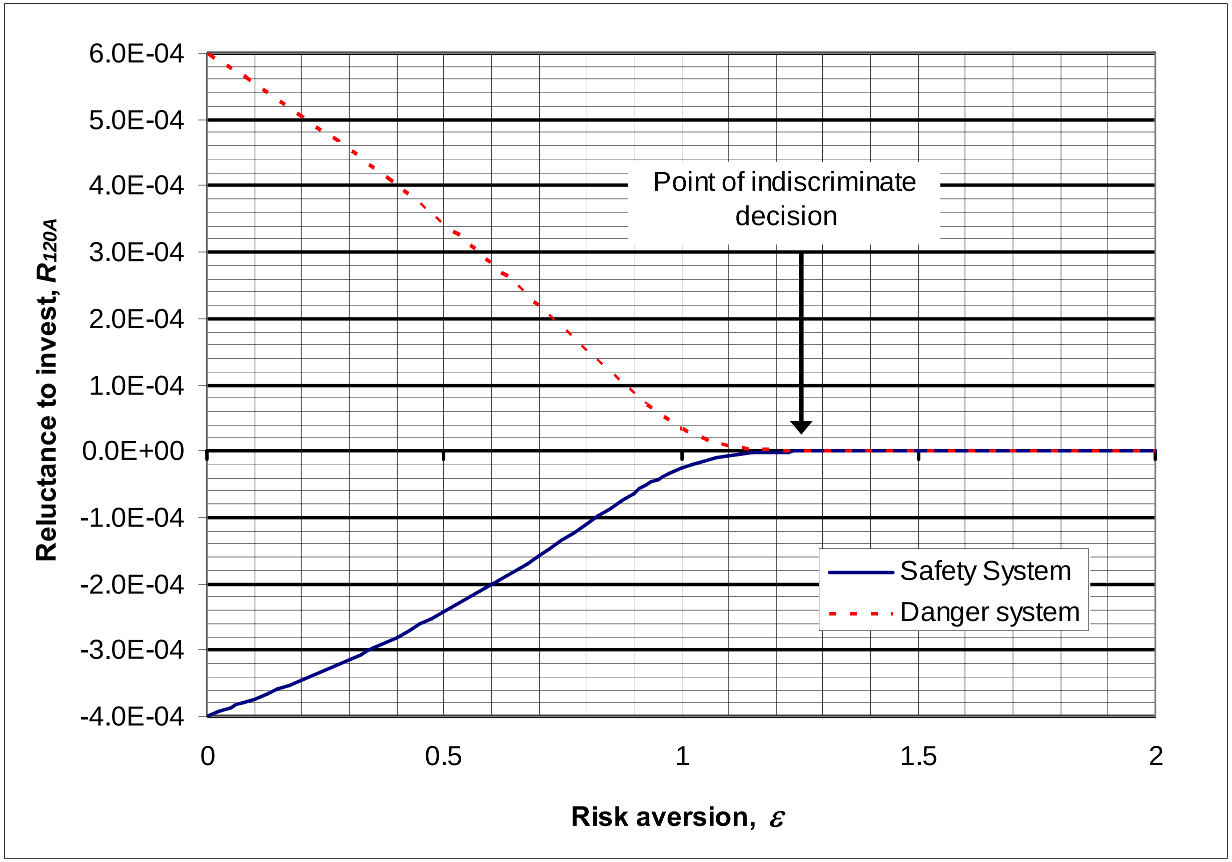
The point of indiscriminate decision



The point of indiscriminate decision

- The point of indiscriminate decision is taken, conservatively, to be the value of risk-aversion that causes the reluctance to invest to have an absolute value less than 10^{-6}.
- The associated risk-aversion, ε_{max}, is the maximum sensible value it can be.

Reluctance to invest in protection system, R_{120A}

$$R_{120A}(\varepsilon, A) = \frac{D(u_1, u_2 | \varepsilon)}{u_0(\varepsilon)}$$

where: $$D(u_1, u_2 | \varepsilon) = E(u_1) - E(u_2)$$

$$u(x) = U_\varepsilon(x) = \frac{x^{1-\varepsilon} - 1}{1-\varepsilon} \qquad \text{for } \varepsilon \geq 0 \text{ but } \varepsilon \neq 1$$

$$= \log x \qquad \text{for } \varepsilon = 1$$

Scale of reluctance to invest

- A 100% reluctance to invest in a protection scheme will occur when the organisation can expect to lose 100% of the utility of its assets as a result.

Risk Multipliers for £10bn organisation

Cost of accident	Frequency of accident in 50-year operating period	Probability of accident before scheme	Probability of accident after scheme	Expected loss before scheme, δZ_0	Maximum risk multiplier. $M_R(\varepsilon_{\max})$	Fair cost of scheme, δZ_R
£5bn	2 x 10^{-5}	10^{-3}	0	£5M	1.34	£6.7M
£0.95bn	2 x 10^{-4}	10^{-2}	0	£9.5M	1.04	£9.84M
£9.5bn	2 x 10^{-5}	10^{-3}	0	£9.5M	3.81	£36.18M

Comments

- The J_2-value approach has resulted in a mathematical model of how a fair decision maker would weigh the possible costs of a major accident against the price of a protection system to prevent them.

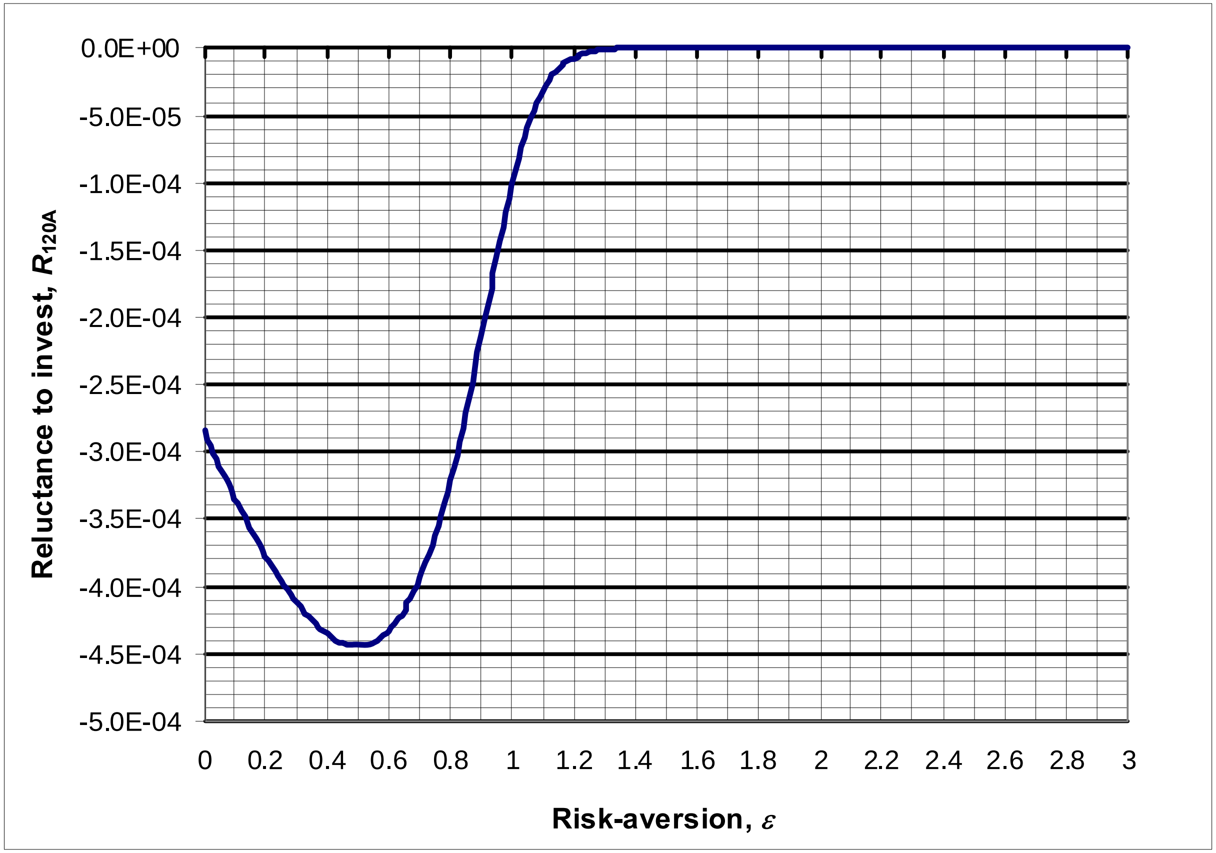
Comments

- The following new concepts have been introduced and defined mathematically:
  - a dimensionless variable, the **reluctance to invest**, with a 100% reluctance associated with a scheme that would absorb the utility of all the organisation's assets
  - the **permission point**, the value of risk-aversion at which the decision to invest in a protection scheme is made
  - the **point of indiscriminate decision** - the highest value of risk-aversion at which a decision to invest can be made, after which the decision maker will be unable to distinguish between good and bad schemes.

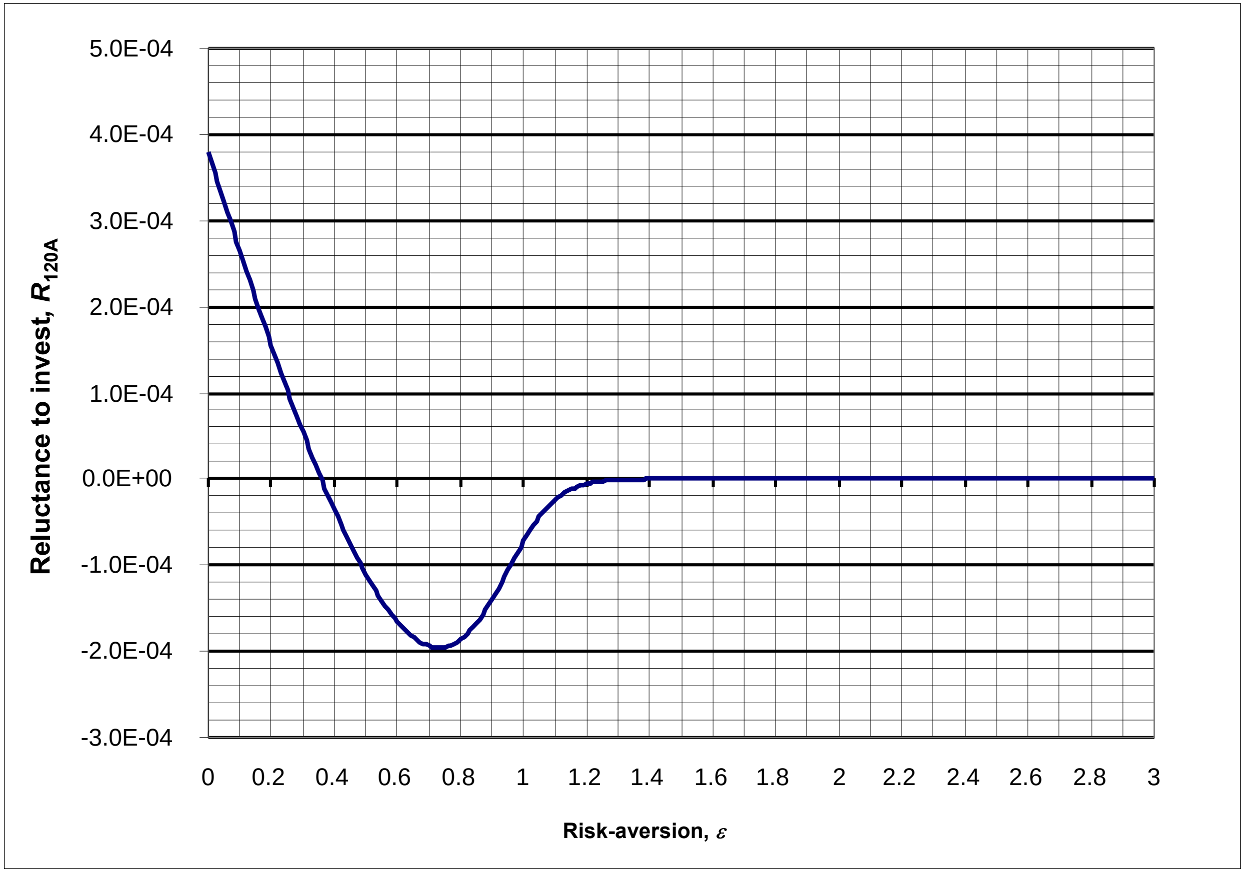
- In addition, the concepts of **disproportion** and **gross disproportion** have been given a mathematical definition and justification.

- Moreover, a mathematical model has emerged of the way a rational, risk-averse decision maker will act in deciding to sanction a protection system:

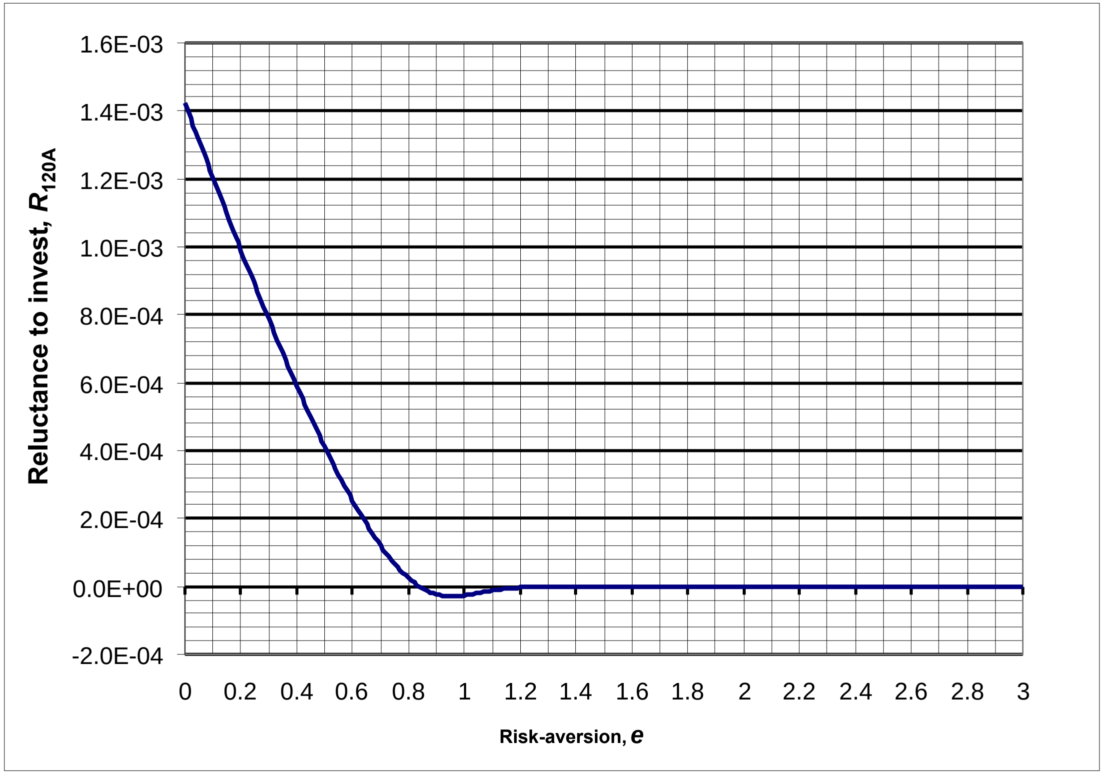
    He mulls the problem over from a risk averse viewpoint, and takes the decision when his risk-aversion value minimises his reluctance/maximises his desire to invest, provided he can still discriminate the advantages of doing so.



Organisation with £10 bn assets, facing accident of probability 10^{-3} and cost £9.5 bn.
Scheme to prevent accident costs 70% of expected monetary saving



Organisation with £10 bn assets, facing accident of probability 10^{-3} and cost £9.5 bn. Scheme to prevent accident costs 140% of expected monetary saving



Organisation with £10 bn assets, facing accident of probability 10^{-3} and cost £9.5 bn. Scheme to prevent accident costs 250% of expected monetary saving

Estimating an average risk-aversion for UK adult

- The decision making model leads on to a way of estimating the risk-aversion of the average UK adult.
- The figures are very much in line with the range of other, recent estimates for a single value for the UK, and provide an explanation for why differences are likely to occur.

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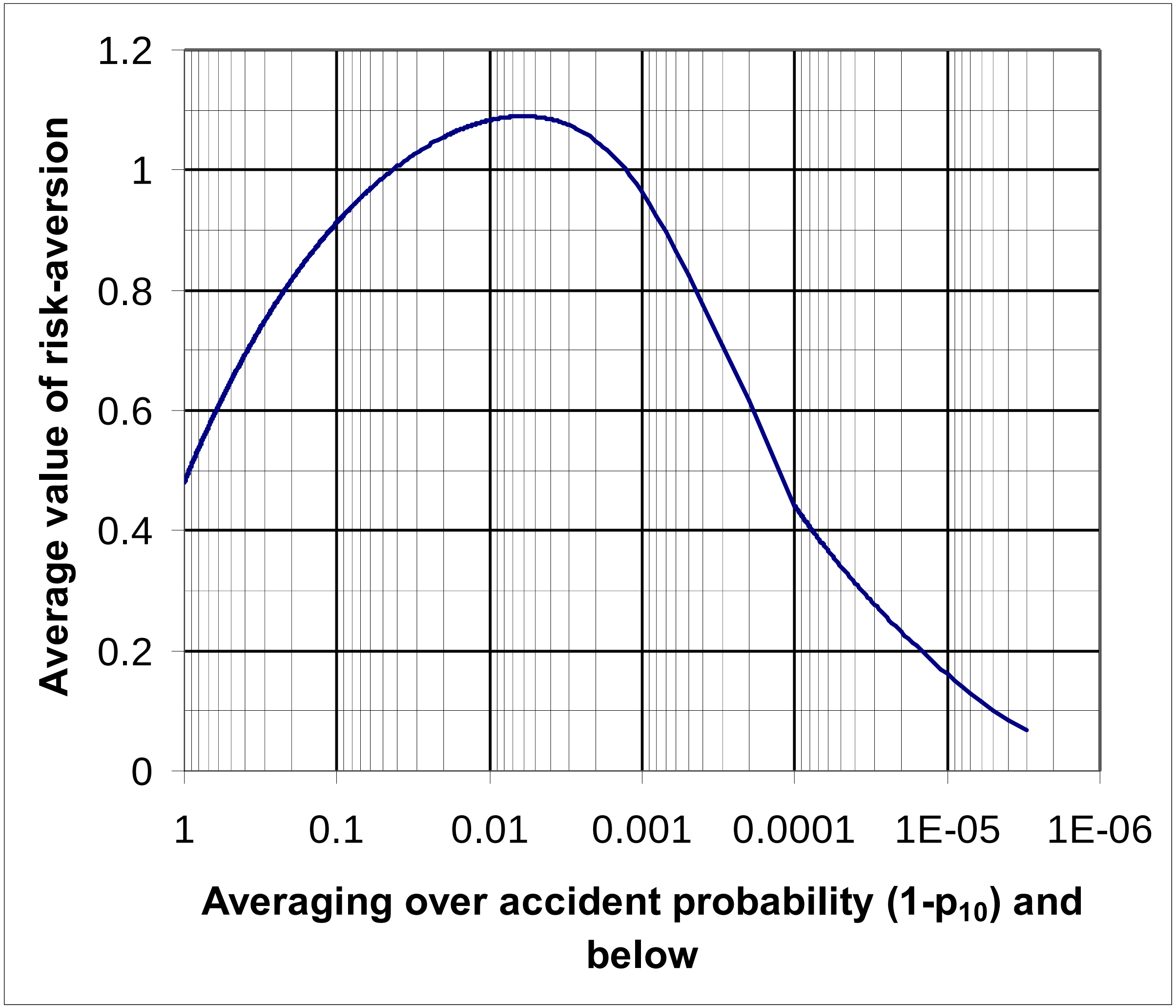

Average value of risk-aversion over all decision space, defined by:

- accident/ loss probability
- extent of loss as fraction of all assets
- cost of protection up to point of indiscriminate decision

Why does the J-value value human life at about twice the Department of Transport figure?

- The DoT figure of about £1.6M per human life is about half the value derived using the J-value.
- The average value of risk-aversion is 0.65 for a risk-averse UK adult who avoids any decision that is more likely to fail than succeed.
- Using $\varepsilon = 0.65$ instead of life risk-aversion, $\varepsilon = 0.82$, in the J-value analysis gives the value of a human life as £1.35M
- The use of a lower effective value of ε provides a reason why the DoT figure might be lower.
- The J-value seems to be building in a disproportion factor of about 2.

Summary

- The J-value technique can be used to assess schemes to protect against human harm.
- The J_2-value can be used to assess schemes guarding against environmental (and other) costs, but no effect on human harm.
- The J_T-value technique can be used to assess schemes to protect against human harm and environmental and other costs.

Advantages of the J-value Framework

- The J-value, the J_2-value and the J_T-value are each **entirely objective.**
- The J_T-value offers a **complete and objective solution** for advice to the decision maker on how much should be spent on a protection system to guard against human harm and environmental and all other costs.

Final thoughts

The J-value framework avoids overspending and underspending.

The J-value framework leads to correct spending on human safety and environmental protection.